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Linear Programming Problem for Allocating Children to Secondary Schools

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ABSTRACT:

The secondary allocation scheme proposed for Reading by Berkshire County Council (B.C.C.) in 1978 has been investigated under the Race Relations Act. After reviewing previous programming studies of devising school catchment areas, this paper uses the technique of goal programming to analyze the problem of proposing catchment areas for the secondary schools in Greater Reading. Six goals are identified: distance' difficulty of journey, racial balance, reading-age retarded balance, sex balance and capacity utilization. The problem is solved for 10 alternative sets of goal weights, and it is shown that most of these dominate the B.C.C. solution.

INTRODUCTION:

Most previous studies of the problem of assigning children to schools have used linear programming methods and have been concerned with American applications. This study uses the more general goal-programming approach to analyze the problem of assigning children to secondary schools in Reading. England. The impetus for this study was a new allocation or zoning system proposed for Reading in 1978 by the local education authority, Berkshire County Council (B.C.C.), which resulted in an enquiry by the Commission for Racial Equality for alleged racial discrimination under the Race Relations Act 1976 (Tufo et al ' and Commission for Racial Equality2).

This study is the first to consider the difficulty of the journey to school as well as the distance. It is also the first to consider the sex and ability of the children being assigned. After briefly reviewing previous linear-programming and goal-programming studies, the model to be used in this study is presented. This model is then applied to the Greater Reading area, and 10 solution corresponding to 10 different sets of goal weights are calculated and tabulated. It is then argued that, if the definition of the problem as encapsulated in the model is accepted, it is possible to generate solutions that dominate the 1978 B.C.C. solution.

LINEAR PROGRAMMING STUDIES:

A typical linear programming (L.P.) formulation of the problem of allocating children to schools is set out below. In following the previous L.P. literature, which is almost entirely American, this model gives considerable prominence to racial groups. This is because the U. S policy of desegregating schools and bussing children to achieve integration is reflected in the way the American authors have set up the problem. Later, when a model is formulated for the U.K., racial balance remains but is given much less importance. Not W is racial balance included as an objective rather than a binding constraint, but other objectives which are included are given greater weight. Furthermore, the separate allocation of different racial groups is not permitted, as it is in the U.S. models.

$$\mathrm{Min}\sum_{i}^{I}\sum_{j}^{J}\sum_{k}^{K} d_{ij} X_{ijk} \qquad (1)$$

subject to:

$$\sum_{i}^{I} X_{ijk} - \sum_{k}^{K}\sum_{i}^{I} X_{ijk}, u \le 0,\ j = 1,\ldots\ldots\ldots n \qquad (2)$$

$$\sum_{i}^{I} X_{ijk} - \sum_{k}^{K}\sum_{i}^{I} X_{ijk}, l \ge 0,\ j = 1,\ldots\ldots\ldots n \qquad (3)$$

$$\sum_{k}^{K}\sum_{i}^{I} X_{ijk} \le c_{J},\ j = 1,\ldots\ldots\ldots n \qquad (4)$$

$$\sum_{k}^{K}\sum_{i}^{I} X_{ijk} \ge m_{j},\ j = 1,\ldots\ldots\ldots n \qquad (5)$$

$$\sum_{j}^{n} X_{ijk} = S_{jk}\, i = 1\ldots\ldots t, k = 1\ldots\ldots\ldots K \qquad (6)$$

$$X_{ijk} \ge 0, i = 1\ldots\ldots t,\ j = 1\ldots\ldots\ldots n, k = 1\ldots\ldots\ldots K \qquad (7)$$

where

xijk = number of children from residential area (tract) i attending school j who are in racial group k;

dij = a measure of distance (or cost) to be travelled between tract i and school j;

u = maximum proportion of children from the minority racial groups in each school; the same proportion applies to each school;

l = minimum proportion of children from the minority racial group in each school; the same proportion applies to each school:

cj = total capacity of school j, i.e. maximum number of children at school j;

m, =minimum number of children at school j :

sik = number of children of racial group k living in tract i;

t = number of tracts;

n = number of schools;

K = number of racial groups.

The objective is to minimize the total distance travelled to school by all children subject to upper and lower racial and capacity constraints and the requirement that each child be allocated to exactly one school.

Where the intake may be less than the full capacity, the racial balance constraint may be expressed in one of two ways. If the racial balance constraint is expressed as I proportion of full capacity, it is equivalent to requiring that the number of ethnic minority children allocated to the school be greater than or less than some specified number of children. The only authors who have used this approach are Clarke and Surkis.4 If the racial balance constraint is expressed as a proportion of the number of children allocated to the school, the number of ethnic minority children who can be allocated to that school will vary with its capacity utilization. This is the approach used by the other authors who have faced this choice and has the advantage that the racial balance constraint a independent of the level of capacity utilization.

This L.P. problem has obvious similarities with the standard transportation (distribution) problem with children as ‘goods', tracts as `warehouses' and schools as outlets. Some previous studies have utilized the transportation algorithm. The L.P. studies of allocations children to schools are summarized in Table 1. The distance between a child's home area (tract) and the school is generally measured by either the road distance or the distance as the crow flies. In some cases the distance measure has been squared in an attempt to ensure compact and contiguous school districts (Weaver and Hess19). Only Liggett has required all the children from a particular tract to be allocated to the same school. Six of the studies permit the separate allocation of the races living in the same tract. None of the L.P. studies considered either the sex or the ability of the child allocated, and only half actually solved their model for a real-world example.

THE GOAL-PROGRAMMING APPROACH

There is a major difficulty associated with the L P approach. It contains two levels of priority with constraints which must be met and an objective function which is considered only after all the constraints have been satisfied. There are many situations in which such an approach is entirely justified, but for the allocation of children to schools, goal programming (G.P.) is a more appropriate technique. The school zoning problem contains not only constraints which must be satisfied but a number of conflicting objectives. For example, it may be thought desirable to minimize the total distance traveled and equalize the racial balance between schools, i.e. there may be multiple and conflicting goals.

G.P. requires the specification of a desired value for each goal (e.g. that the proportion of ethnic minority children in each school is 10%) and a set of weights that is applied to positive and negative deviations from the specified goals. These weights reflect the relative importance of deviations from the corresponding goal. Such a weighting can take one of three forms: (a) the weights are similar orders of magnitude, e.g. $W1=5W2$, so that goal one is regarded as five times more important than goal two; (b) the weights are of different orders of magnitude, e.g. $W1 >>> W2$, where goal one is infinitely more important than goal two (a pre-emptive ordering); (c) a mixture of two types.

The advantage of the pre-emptive ordering approach is that it removes the need to specify relative weights beyond ranking the goals. This approach is appropriate if the underlying

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preferences are lexicographic. However, if preferences are such that trade-offs between goals are possible, the appropriate approach is to quantify these trade-offs. It will be assumed that preferences are such that there is linear trade-off between goals and that there is no interaction between goal deviations, i.e. the objective function is additive.

PREVIOUS GOAL-PROGRAMMING STUDIES

The first G.P. approach to the problem of allocating children to schools was Lee and Moore. Their model contained six goals, which were to minimize:

a. the number of children not allocated to any school

b. deviation from the average racial balance

c. total transport costs

d. the use of road distances above 8 miles

e. deviations from the average level of overcrowding

f. the underutilization of school capacity

There were no conventional constraints in the model, only goals. This means it was possible for some children not be allocated to am school or for the number of children allocated to a school to exceed its capacity. However. Lee and Moore used the pre-emptive G P. method and in each of the three alternative goal rankings considered goal (a) was always the first priority. This effectively gave goal (a) the status of a binding constraint. The model allowed for two racial groups and permitted the different allocation of races wing in the same residential area. The racial balance goal and the overcrowding balance goal were expressed as proportions of the number of children allocated to the school and not as proportions of school capacity. Lee and Moore did not apply their model to a real-world problem but only to a hypothetical example. Since pre-emptive G.P. was used, trade-offs between the various goals were prevented.

A later application of G.P. by Kuntson et al., did permit trade-offs between the various goals. There were three goals, which were to minimize.

(a). deviations from the average racial balance

(b). the extent to which the crow flies distance was below some arbitrary constant

(c). the extent to which the number of pupils assigned to a school was less than 100% of capacity.

In addition, the solution was constrained to allocate each child exactly once to a school. The model contained two races and permitted the different allocation of races living same district. The racial balance goal was expressed as a fixed number of children independent of the capacity utilization of the school. There were no conventional constraints on capacity so there was no upper bound on the number of children that could be allocated to a school. The model was applied to a hypothetical example. Subsequently Saunders applied essentially the same G.P. model to an unnamed real-world example. The only differences were that the third was modified to minimize the extent to which the number of pupils assigned to a than 110% capacity and that the model solutions were subsequently hand-adjusted.

GOAL PROGRAMMING AND SECONDARY ALLOCATION IN READING

The problem facing the Education Committee of B.C.C. in 1978 was to define catchment areas for the 17 comprehensive schools in the Greater Reading area. No doubt similar problems periodically face all local education authorities in the U.K., and the model developed for Reading could equally be applied other areas of the U.K. Greater Reading has 64 mixed primary schools, 11 mixed comprehensive schools, 6 single-sex policy, were ignored for the purpose of this study. No data were available on the home addresses of the 3330 children transferring from primary to secondary schools in September 1979. However, the children attending a particular primary school were generally known to live in the catchment area defined for that primary school. Since the primary school catchment areas were contiguous areas surrounding the school, the location of a child's primary school was taken as a proxy for the child's home address. As there are six single sex secondary schools, the model must permit the separate allocation of the boys and girls at the same primary school. To accomplish this, each primary school was disaggregated into boys and girls to give 128 'schools', with the first 64 'schools' being boys in the 64 primary schools and the 64 'schools' being the girls in the 64 primary schools. This is the first study to use 'schools’ as the unit to be allocated, and this approach besides conforming most closely to the method adopted by education authorities in the U.K. considerably simplifies the collection data. It also means the problem is structured to permit the establishment or maintenance of linkages between a secondary school and a group of feeder primary schools.

The model contains two sets of conventional constraints. The number of children allocated to each secondary school must not exceed the school's capacity and the children w each of the 128 'schools' must be allocated to a secondary school exactly once. There are six goals in the model, which are to minimize:

(a). deviations from the average racial balance.

(b). deviations from the average reading-age retarded proportion.

(c). total road distance travelled,

(d).total difficulty of travel,

(e). deviations from the average capacity utilization,

(f). deviations from the average sex proportions.

The racial-balance goal was to reduce the concentration of the ethnic minority children m the area in a few secondary schools. Children were defined as being reading-age retarded if they had a reading age measured two or more years behind chronological age. The reading –age retarded goal was included to ensure that no secondary school received a disproportionately large number of remedial children, and, within the limitations of the model, secondary school intkes were as far as possible balanced with respect to a simple measure of academic ability. The road-distance goal was to try and ensure that children were allocated to their nearest secondary school. The difficulty of travel goal was included because it was recognized that, owing to the nature of the bus system and rush-hour traffic flows, some journeys of given distance are easier than others. Difficulty of travel was measure by awarding points. In urban areas, one point was awarded for each of the following difficulties (a) walking over half a mile; (b) catching one bus; (c) crossing the town centre. In rural areas, difficulty was scored subjectively on likely congestion, difficulty of route and distance driven through built-up areas. The capacity utilization goal was included to prevent some secondary schools being grossly underused. Since school s are staffed on the basis of a fixed pupil-teacher ratio, a small number of pupils means a small number of teachers, which in turn endangers the maintenance of the curriculum. Finally, the aim of the sex goal is to ensure that the sex balance in the mixed schools is consistent with that in the population of children to be allocated.

The resulting model can be stated as:

$$\begin{aligned} \text{Min}\,\alpha_1\left(\sum_j^n y_j^+ + \sum_j^n y_j^-\right) + \alpha_2\left(\sum_j^n z_j^+ + \sum_j^n z_j^-\right) + \alpha_3\,(q^+ + q^-) + \alpha_4\,(q^+ + q^-) \\ + \alpha_5\left(\sum_j^n V_j^+ + \sum_j^n V_j^-\right) + \alpha_6\left(\sum_j^n w_j^+ + \sum_j^n w_j^-\right) \end{aligned} \quad (8)$$

The decision variable (xij) was permitted to take on values in the zero-range. This implied that the boys from a particular primary school could be split between a number of secondary schools, whilst the girls from that primary school could be split between a different set of secondary schools. In contrast to most previous studies, the various races were not allocated separately nor were the different ability groups. This means that if a school is split, the children allocated to each secondary school are assumed to be chosen to reflect the proportions of ethnic minority and reading age retarded children in the school concerned. A model which treats all children at the same school in the same way was felt to be more acceptable to parents and to the LEA. Discrimination between children on the basis of their racial group may lead to a contravention of the Race Relations Act (1976) and the necessary data on the numbers of children in each school who were both reading age retarded and from an ethnic minority were unavailable.

The problem had one objective function, 145 constraints and 64 goal constraints, making 210 rows in tha data matrix. There were 1792 decision variables (14*28), 128 goals (64*2) and one-right hand side, making 1921 columns. Hence the data matrix contained 403.410 numbers.

SETTING THE GOAL VALUES AND THE GOAL WEIGHTS

Setting the goal values was a straightforward task. The distance and difficulty goals were set equal to zero whilst the ethnic minority, reading age retarded and sex goals were set equal to the proportions in the population of children to be allocated. Finally, the capacity-utilization goal was set equal to the total number of children to be allocated divided by the total number of places available in the 17 secondary schools. The goal values are given in Table 2.

TABLE-2. TARGET GOAL VALUES

Distance	0
Difficulty	0
Ethnic minority	7.4%
Reading-age retarded	10.0%
Proportion of boys	50.1%
Capacity utilization	94.5%

The establishment of a set of goal weights is a far more difficult task involving value judgments about the relative importance of the conflicting goals, as an infinite number of sets of goal weights exists. It was decided first to examine six extreme cases, giving all the weight to each goal in turn. This would establish the minimum possible value for each goal, so setting a reference point for judging other weighting schemes. It also meant that the distance minimizing solution would be computed, thereby including the traditional L.P. approach as a special case.

In establishing set of plausible weights for the six goals, the problem of different goals being measured in non-comparable units had to be faced. The six goals could be divided into three groups.

(a). the distance goal measured in miles,

(b). the difficulty goal, which was a dummy variable,

(c). the ethnic minority, reading age retarded, sex and capacity-utilization goals, which were all measured in terms of numbers of children.

It was easier to establish relative weights within group (c) were measured in the same units.

Another factor relevant to setting goal weights was the degree of correlation between various goals. The distance and difficulty goals are both concerned with the case of the journey from home to secondary school. Data were available on the distance and the difficulty of 1088 potential journeys, and the correlation between the distance and difficulty was 0.66. This indicates that whilst there was sufficient difference between these two variables to justify including both, the solution to the GP model will not be very sensitive to varying the relative weights on the distance and difficulty goals. For each f the 128 schools data were available on the number of ethnic minority and reading age retarded children. The correlation between these two numbers was 0.38, showing there was a substantial difference between these variables and supporting their inclusion as separate goals. It also indicates that the solution may be sensitive to changes in the relative weights on the ethnic minority and reading age retarded goals.

The American studies have given considerable prominence to the proportion of ethnic minority children in each school. Nine of the LP models have included radial-balance constraints, whilst both GP models have included racial balance goals. This may be due to US legislation on the desegregation of schools and to the use of bussing to achieve racial balance. However, in the context of reading, racial imbalances per se were not considered to be of great importance. The 10 sets of goal weights as used in this study are set out in Table-3.

No	Goal weights					
	Distance	Difficulty	Reading age retarded	Ethnic minority	Proportion of boys	Capacity utilization
1	100	0	0	0	0	0
2	0	100	0	0	0	0
3	0	0	100	0	0	0
4	0	0	0	100	0	0
5	0	0	0	0	100	0
6	0	0	0	0	0	100
7	18	18	36	12	4	12
8	15	15	39	13	3	15
9	19	15	45	0	4	17
10	12	12	45	13	3	15

The first six rows are simply those where, in turn, each goal overrides all others to reveal the solution yielding for each the minimum possible value. Rows 7-10 are goal weights selected to represent appropriate values and to test for the trade off between goals in the value of the solution obtained.

As noted above, there is no uniquely appropriate set of weights. The weights reflect value judgments. There is, in addition, the problem of scaling as between distance and difficulty weights on the one hand and reading retarded, ethnic sex-mix and capacity constraints on the other. The values shown in Table-3 are chosen partly on the basis of a priori judgments that, in the case of reading, distance of journey was somewhat more important than the journey's difficulty and the reading retarded mix was considerably more important than the ethnic or sex mix. The choice of goal weights was also conditioned by aspects of the results produced in successive runs and reported in Tale-4, in particular the trade-off these results suggested as between conflicting goals. Thus one reason for the low weight on sex mix is the comparative case with which that goal could be nearly met.

The results are formally discussed in the next section. Since the goal weights were arbitrarily constrained to sum to 100, it was not possible to change one goal weight in weight alone from the results presented in Table-4. However, since the weights for formulations 7-10 are of broadly similar magnitude, crude comparisons are possible. For example, in formulation 9, in which the weight on the ethnic minority goal was reduced to zero, the main effect was a clear improvement in the sex ratio and, as expected a deterioration in the ethnic minority goal. The other goals remained largely unchanged.

RESULTS

The model was solved 10 times, once for each set of goals weights, using ICLLINPROG on an ICL 1904S. The total computer CPU time required was 18 hours 22 minutes. For comparison, the problem was also solved on a larger computer with a more modern LP package, a CDC 7600 machine using the APEX3 package. The total CPU time required by the CDC was approximately seven minutes, i.e about 165 times faster than the ICL machine.

Table-4. Deviations from target goal values for alternative sets of goal weights

Set of goal weights	**Deviations from target goal weights**						
	Distance in miles	Difficulty	Reading-age retarded	Ethnic minority	Proportion of boys	Capacity utilization	Maximum distance in miles
1	3859	3473	92.1	207.2	30.5	243.9	5.6
2	5116	3033	82.8	153.3	3.2	228.0	11.3
3	4747	3569	0	109.4	17.8	94.8	7.9
4	4836	3500	70.6	0	4.0	142.8	5.8
5	4097	3282	81.3	190.5	0	117.2	5.6
6	4252	3342	87.1	201.6	4.1	0	5.6
7	4042	3265	81.8	195.3	13.0	157.3	5.6
8	4099	3283	80.5	193.6	18.1	95.8	5.6
9	4041	3323	78.7	211.8	5.2	117.9	5.6
10	4138	3316	58.5	183.0	35.6	99.8	5.6
BCC 1978	4813	3657	120.4	211.9	28.0	270.6	5.6

For the first time six sets of goal weights, when deviations from a single goal were minimized, it was suspected there would be multiple solutions. So an attempt was made to ensure that, if this was the case, the solution selected from amongst the multiple solutions was reasonably sensible (in terms of the other five goals). This was done by giving the goal being considered a weight of 100 and the other five goals a weight of 0.00001. The suspicions of multiple solutions were given limited support by an analysis of the dual values for variables not in the optimal solution. If such a value is very small, it indicates that there would be very little effect on the value of the objective function by including this variable in the optimal solution (and removing some other variable). The average number of dual values for non-basic variables that were smaller than 0.000001 was 3.3, and it is clear that whilst the problem of multiple solutions is present, it is not severe.

The deviations from target goal values for the 10 sets of goal weights considered are set out in Table 4, along with the 1978 BCC zoning scheme. This table shows that it is impossible to devise a zoning scheme that involves children traveling less than 3859 miles or incurring 3033 units of difficulty. However, it is possible to produce zoning schemes that involve no deviations from the other four goals.

DISCUSSION OF THE RESULTS

When all goals were given significant positive weights, the values for distance and difficulty remained within 10% of their minima. This compares with an excess over their minima (Table 4, rows 1 and 2) in the BCC solution of 24.7 and 20.6% respectively. It can be seen from Table 4 that the zoning schemes produced by the sets of goal weights 3,5,6,7,8 and 9 dominate the 1978 BCC scheme. That is, they produce a lower deviation from the value set for every goal than does the 1978 BCC scheme. Furthermore, for four goals (difficulty, reading age retarded, ethnic minority and capacity utilization), the deviations from target goal values for the 1978 BCC scheme exceed those of all of the 10 weighting schemes analyzed. This includes six sets of goal weights that give no weight to reading age retarded, capacity utilization and difficulty and seven that given no weight to ethnic minorities.

In fact, BCC appear to have imposed two additional requirements on their 1978 solution. First, their solution is largely integer, with all the boys or girls from a particular primary school being allocated to the same secondary school. Almost 93% of the schools were allocated entirely to a single secondary school by BCC scheme, whilst the corresponding average across the 10 solutions generated using GP was 82%. If only the more plausible sets of goal weights are considered, i.e. the last four, this figures rises to over 85%. Second, they have usually allocated the boys and girls from a particular primary school to the same mixed comprehensive. Boys and girls are of course allocated separately to single-sex comprehensives. However, as well as imposing additional conditions, the 1978 BCC solution also relaxes some of the constraints used in this study. The 1978 BCC solution violates three of the capacity constraints imposed in this study, in one case by 25.1% (School N). An analysis of the dual values associated with the 17 capacity constraints revealed that these capacity constraints were generally only of importance for five schools (including School N). Since the capacity constraint for school N was binding on a number of occasions the optimal solutions presented in table-4 (excluding the 1978 BCC solution) would have been improved if this school had been allowed to exceed its capacity by as much as 25.1%.

Table 4 also contains the maximum distance between any primary school and the secondary school to which some or all of its pupils are allocated. As can be seen, this figure is generally 5.6miles, the same as for the 1978 BCC solution. This figure is of interest because an optimal allocation may involve some children traveling very long distances if this is offset by other children traveling only a short distance. Some consideration was give to imposing an upper

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bound on the distance between any child's primary and secondary schools. However, Table 4 shows that, except for the second and third sets of goal weights, this is not a problem.

This potential difficulty of the solution being optimal when aggregated across all pupils but inequitable between pupils was investigated further by calculating the weighted sum of the deviations from target goal values for each secondary school. These numbers (adjusted for differences in the numbers of children allocated to each school) were used to compute an index number for each school for the solutions to the last four sets of goal weights. The average value of this index is 100, with values below 100 indicating that, using the relevant set of goal weights, the children allocated to that school are closer to the target goal values than are children allocated to a school where the value of the goal index is above 100. These goal index numbers for each secondary school remain more or less constant, despite the variation in the goal weights. Schools D, G, H and Q have consistently low numbers, whilst Schools A and K have consistently high values. It appears that these inequalities are largely due to the distance which pupils must have travel to reach some more rural schools.

CONCLUSIONS

The secondary allocation polices presented in this paper can either be regarded as a solution applicable to 1979 only, or as a solution that is also suitable for future years. However, these polices can only be regarded as a solution applicable to future years if there is little change in the relevant features of the Reading school system. And even if the allocation policy is only regarded as applying to 1979, some further assumptions must be met. The analysis has concentrated on various aspects of the 1979 age cohort entering each secondary school. It has not considered the effect which the 1979 entry has on the racial, sexual and remedial balance of each secondary school as a whole nor on its capacity utilization. If the goals are regarded as applicable only to age cohorts, this widening of the problem is not necessary. But if the characteristics of the whole secondary school are also of importance and if the relevant features of the situation are changing over time, the allocation problem will have to be generalized. Thus it is only legitimate to consider the allocation of the 1979 cohort in isolation if either:

(a). there are no changes over time in the relevant aspects of Reading's educational system.

(b). the solution applies to 1979 only no importance is attached to the effects which the entry of this cohort will have upon the nature of the secondary school concerned.

This paper has argued that GP which permits trade-offs between the goals is superior to traditional approaches to the school zoning problem. It has been shown how such a GP model incorporating six goals can be formulated and solved. Whilst the optimal solution obtained depends upon the set of goal weights used, it has been found to be a straightforward matter to produce solutions that dominate the 1978 BCC solution, although the BCC in devising their scheme, may have a different view of the problem from that used here. Finally, various assumptions must be met before it is valid to devise a zoning scheme for one age cohort in solution.

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